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Circular semiclassical string solutions on confining AdS/CFT backgrounds

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Abstract: We study multiwrapped circular string pulsating in the radial direction of AdS black hole. We compute the energy of this string as a function of a large quantum number n. One then could associate it with energy and a quantum number of states in the dual finite temperature $\mathcal{N} = 4$ SYM theory as well as three-dimensional pure gauge theory. We observe that the n dependence of the energy has a universal form. We have also considered pulsating string in the background of the near-extremal D4-brane solution. Circular pulsating membrane in M-theory on $AdS_7 \times S^4$ has also been studied.

Keywords: Penrose limit and pp-wave background, AdS/CFT and dS/CFT Correspondence.

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Contents

1. Introduction

It is conjectured that type-IIB string theory on $AdS_5 \times S^5$ with N fluxes on the S^5 is dual to four-dimensional $\mathcal{N} = 4$ SU(N) SYM theory [1, 2, 3]. According to this conjecture the spectrum of single string states on $AdS_5 \times S^5$ corresponds to the spectrum of single trace operators of the $\mathcal{N} = 4$ gauge theory. However, until recently, this correspondence had only been studied for supergravity modes on $AdS_5 \times S^5$ which are in one-to-one correspondence with the chiral operators of the $\mathcal{N} = 4$ gauge theory.

In an interesting recent development the authors of [4] have been able to study the AdS/CFT correspondence beyond the gravity modes. In fact it has been conjectured [4] that the string theory on the maximally supersymmetric ten-dimensional PP-wave has a description in terms of a certain subsector of the large-N four-dimensional $\mathcal{N} = 4$ SU(N) supersymmetric gauge theory at weak coupling. More precisely this subsector is parametrized by states with conformal weight Δ carrying J units of charge under the U(1) subgroup of the SU$(4)_R$ R-symmetry of the gauge theory, such that both Δ and J are parametrically large in the large 't Hooft coupling while their difference, $\Delta - J$ is finite. Therefore it has been possible to work out the perturbative string spectrum from the gauge theory side.

Shortly after, the authors of [5] identified certain classical solutions representing highly excited string states carrying large angular momentum in the AdS_5 part of the metric with gauge theory operators with high spin S and conformal dimension Δ which is identified with the classical energy of the solution in the global AdS coordinates. An interesting observation of [5] is that the classical energy of the rotating string in AdS_5 space in the limit of $S \gg \sqrt{\lambda}$ scales as

$$\Delta - S = \frac{\sqrt{\lambda}}{\pi} \ln \frac{S}{\sqrt{\lambda}} + \cdots \,, \tag{1.1}$$

which looks like logarithmic growth of anomalous dimensions of operators with spin in the gauge theory. It has also been shown that the BMN operators [4] can also be identified with classical solutions of string in AdS_5 with angular momentum in S^5 space [5].

A generalization for the case where the string is stretched along radial direction of AdS and rotates along both AdS_5 and S^5 spaces has also been studied in [6]. This solution corresponds to those operators which have both spin and R-charge in the gauge theory side. A more general solution where the string is also stretched in an angular coordinate of S^5 has been studied in [7]. For further study in this direction see [8]–[14].

Recently another semiclassical string solution on $AdS_5 \times S^5$ has been studied [15]. This string configuration corresponds to a multiwrapped circular string pulsating in the radial direction of AdS_5 and fixed to a point on S^5. More precisely in the notation in which the AdS metric is written as

$$\begin{aligned} ds^2 &= \alpha' \lambda^{1/2}(-\cosh^2 \rho\, dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2)\,, \\ d\Omega_3^2 &= \cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2\,, \end{aligned} \tag{1.2}$$

the string solution is given by

$$t = \tau\,, \qquad \rho = \rho(\tau)\,, \qquad \phi = m\sigma\,, \qquad \theta = \frac{\pi}{2}\,, \tag{1.3}$$

where σ and τ are worldsheet coordinates.

For the string with large values of energy, one can use the Bohr-Sommerfeld analysis to find the energy of the string as a function of a large quantum number, the excitation level of the string, as following:[1]

$$E \approx 2n + \frac{8\pi^{1/2}}{\Gamma(\frac{1}{2})^2} \lambda^{1/4} \sqrt{mn}\,. \tag{1.4}$$

Using the AdS/CFT correspondence one can relate the quantum number to the dimension of an operator in the $\mathcal{N} = 4$ SYM theory. From (1.4) one can read the anomalous dimension of the corresponding gauge theory operator [15]

$$\Delta \sim \Delta_0 + \lambda^{1/4} \sqrt{mn}\,, \tag{1.5}$$

where $\Delta_0 = 2n$ is the bare dimension of the operator and mn corresponds to the string level.

One can also consider a circular string pulsating in the S^5. In this case the energy as a function of a large quantum number is given by [15]

$$\Delta \sim \Delta_0 + \lambda \frac{m^2}{2n}\,. \tag{1.6}$$

The aim of this article is to generalize this string solution to the AdS black hole background. Since this background is conjectured to be dual to a confining gauge theory, this might increase our knowledge about this theory and thereby QCD.

[1] The leading term in this expression was first obtained in [16].

The paper is organized as follows. In section 2 we shall consider circular pulsating string in AdS_5 black hole. Using WKB approximation we will compute the energy of string in terms of a large quantum number. This string could be identified with some state in finite temperature $\mathcal{N}=4$ SYM theory. We also observe that the energy dependence of n has a universal form for $\mathcal{N}=4$ SYM theory and its deformations. In section 3 we will study circular pulsating string in the background of near-extremal D4-brane solution in the decoupling limit. This is supposed to be dual to a four-dimensional pure gauge theory (QCD) [20].[2] In section 4 we shall study circular pulsating membrane solution in M-theory on $AdS_7 \times S^4$. The last section is devoted to conclusions.

2. Circular string in confining gravity background

In this section following [15] we shall study the circular string in the background which describes confining theories. In the AdS/CFT context these backgrounds have the following typical form:

$$ds^2 = f(r)\, dM_d^2 + dr^2 + dN_{9-d}^2\,, \tag{2.1}$$

where dM_d^2 is the Minkowski metric of R^d where the gauge theory lives, r is holographic radial coordinate and dN_{9-d}^2 is the internal space of full ten-dimensional superstring theory.

The gauge theory has confinement phase if $f(r)$ has a local minimum at, say, $r=r_*$ with $f(r_*)>0$ (for detail see for example [17]). In this case the Wilson loop computation leads to a linear potential for quark-anitquark and the effective string tension is given by

$$T_0 = \frac{f(r_*)}{2\pi\alpha'}\,. \tag{2.2}$$

In this section we study those deformed AdS_5 gravity backgrounds which exhibit this property.

2.1 Finite temperature

Let us first consider the finite temperature $\mathcal{N}=4$ SYM theory. The gravity dual of this theory is given by AdS black hole with the metric [18, 19, 20]

$$\begin{aligned} ds^2 &= -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2 d\Omega_3^2 + R^2 d\Omega_5^2\,, \\ d\Omega_3^2 &= d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2\,, \end{aligned} \tag{2.3}$$

with

$$f(r) = 1 + \frac{r^2}{R^2} - \frac{M^2}{r^2}\,, \tag{2.4}$$

where M is related to the black hole mass and for high-temperature we have $M^2/R^2 \sim (RT)^4$ with T being the temperature [20]. This metric is asymptotic to AdS_5 in global coordinates and the boundary of this gravity solution where the gauge theory lives is $S^1 \times S^3$. This solution provides gravity description of the finite temperature of $\mathcal{N}=4$ SYM theory on $S^1 \times S^3$.

[2] We note, however that at low energies this theory has more states with the same mass than pure gauge theory in four dimensions [29].

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Semiclassical description of rotating strings in this background has been studied in [9]. Now we would like to study the semiclassical quantization of a circular string that expands and contracts in this background. In other words we shall consider a pulsating string which is wrapped m times around the ϕ direction sitting at $\theta = \pi/2$. The string configuration is given by

$$t = \tau\,, \qquad r = r(\tau)\,, \qquad \phi = m\sigma\,, \qquad \theta = \frac{\pi}{2}\,, \tag{2.5}$$

where the string worldsheet is parameterized by τ and σ.

The Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-\det(G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)} \tag{2.6}$$

for the configuration (2.5) reads

$$S = -\frac{m}{\alpha'}\int dt\,\left(\frac{r^2}{f}\right)^{1/2}\sqrt{f^2 - \dot{r}^2}\,. \tag{2.7}$$

Here dot represents derivative with respect to t. By making use of the following change of variable:

$$\xi = \int \frac{dr}{f(r)}\,, \tag{2.8}$$

the action (2.7) can be recast to

$$S = -\frac{m}{\alpha'}\int dt\, g(\xi)\sqrt{1-\dot{\xi}^2}\,, \tag{2.9}$$

where $g(\xi) = \sqrt{r^2 f}$. Treating this action as a one-dimensional quantum mechanical system, one finds the hamiltonian of the theory as following:

$$H = \sqrt{\Pi^2 + \left(\frac{m}{\alpha'}\right)^2 g(\xi)^2}\,, \tag{2.10}$$

with

$$\Pi = \frac{m}{\alpha'}\,g(\xi)^2\frac{\dot{\xi}}{\sqrt{1-\dot{\xi}^2}} \tag{2.11}$$

being the canonical momentum. Note that H^2 can be considered as a one-dimensional quantum mechanical system with potential

$$V(\xi) = \left(\frac{m}{\alpha'}\right) g(\xi)^2\,. \tag{2.12}$$

Therefore, we can use the Bohr-Sommerfeld analysis for the quantization of the states. The quantization condition is given by

$$\left(n+\frac{1}{2}\right)\pi = \int_{\xi_1}^{\xi_2} d\xi\,\sqrt{E^2 - \left(\frac{m}{\alpha'}\right)^2 g(\xi)^2}\,, \tag{2.13}$$

where $\xi_{1,2}$ are the turning points.

It is useful to return to the original coordinate r in which the quantization condition becomes

$$\left(n+\frac{1}{2}\right)\pi = E\left[\int_{r_H}^{r_1}\frac{dr}{1+\frac{r^2}{R^2}-\frac{M^2}{r^2}} - \int_{r_H}^{r_1} dr\frac{1-\sqrt{1-\frac{1}{B^2}\left(r^2+\frac{r^4}{R^2}-M^2\right)}}{1+\frac{r^2}{R^2}-\frac{M^2}{r^2}}\right], \qquad (2.14)$$

where $B=\alpha' E/m$ and

$$r_1 = \frac{R}{\sqrt{2}}\left(\sqrt{1+\frac{4(B^2+M^2)}{R^2}}-1\right)^{1/2},$$
$$r_H = \frac{R}{\sqrt{2}}\left(\sqrt{1+\frac{4M^2}{R^2}}-1\right)^{1/2}. \qquad (2.15)$$

For large B ($B/R \gg 1$) the first integral in (2.14) becomes

$$\int_{r_H}^{r_1}\frac{dr}{1+\frac{r^2}{R^2}-\frac{M^2}{r^2}} \approx \frac{\pi}{2}\sqrt{\frac{R^3}{M}}-\sqrt{\frac{R^3}{B}}, \qquad (2.16)$$

while for second integral, setting $r=y\sqrt{BR}$, one finds

$$\int_{r_H}^{r_1} dr\frac{1-\sqrt{1-\frac{1}{B^2}\left(r^2+\frac{r^4}{R^2}-M^2\right)}}{1+\frac{r^2}{R^2}-\frac{M^2}{r^2}} \xrightarrow{\text{large } B} \sqrt{\frac{R^3}{B}}\int_0^1\frac{dy}{y^2}\left(1-\sqrt{1-y^4}\right) =$$
$$= \sqrt{\frac{R^3}{B}}\left(-1+\frac{(2\pi)^{3/2}}{\Gamma(\frac{1}{4})^2}\right). \qquad (2.17)$$

Thus altogether we get

$$\left(n+\frac{1}{2}\right)\pi \approx R\left[\frac{\pi}{2}\left(\frac{R}{M}\right)^{1/2}E - \frac{(2\pi)^{3/2}}{\Gamma(\frac{1}{4})^2}\left(\frac{MR}{\alpha'}\right)^{1/2}\sqrt{E}\right], \qquad (2.18)$$

which can be inverted to find energy as a function of n

$$ER \approx \left(\frac{4M}{R}\right)^{1/2} n + \frac{4\sqrt{\pi}}{\Gamma(1/4)^2}\sqrt{\frac{4MR}{\alpha'}}\left(\frac{4M}{R}\right)^{1/4}\sqrt{mn}\,. \qquad (2.19)$$

As we see the n dependence of energy is the same as that in the conformal case (1.4), of course, up to a numerical coefficient. We note, however, its interpretation from finite temperature $\mathcal{N}=4$ SYM theory point of view might be different. This is because the notion of anomalous dimension is defined only at or near a conformal point. Nevertheless this expression can be thought of as the dispersion relation of stationary states in the gauge theory side.

2.2 Witten's confining model

Starting from the finite temperature solution (2.3) for large M one can use a change of variable which reduces the solution (2.3) to a solution with boundary $R^3 \times S^1$. This solution would provide gravity description of three-dimensional gauge theory which exhibits confinement. The resulting solution is the same as that obtained by scaling of the near-extremal brane solution [21, 22].

For the model we are considering the corresponding solution is obtained from near-extremal D3-brane solution which is given by

$$ds^2 = r^2\left(R^2\, h(r)\, d\phi^2 + dy^2 - dt^2\right) + \frac{dr^2}{r^2 h(r)} + d\Omega_5^2\,, \tag{2.20}$$

where $h(r) = 1 - (r_0/r)^4$.

The rotating string in this background has recently been studied in [13]. We would like to study the pulsating string in this background which is also wrapped around ϕ direction. This string configuration is given by

$$t = \tau\,, \qquad r = r(\tau)\,, \qquad \phi = m\sigma\,. \tag{2.21}$$

The Nambu-Goto action (2.7) for this solution reads

$$S = \frac{-mR}{\alpha'}\int dt\sqrt{h\, r^4 - \dot{r}^2}\,. \tag{2.22}$$

Defining a new variable, ξ, such that $dr/d\xi = \sqrt{r^4 - r_0^4}$, and using the same procedure as the previous subsection, we can write the hamiltonian in the following form:

$$H = \sqrt{\Pi^2 + \left(\frac{mR}{\alpha'}\right)^2\left(\frac{dr}{d\xi}\right)^2}\,. \tag{2.23}$$

One can now use the Bohr-Sommerfeld analysis for the quantization of the states. The quantization condition in the original r coordinate is given by

$$\left(n + \frac{1}{2}\right)\pi = E\left[\int_{r_0}^{r_1}\frac{dr}{\sqrt{r^4 - r_0^4}} - \int_{r_0}^{r_1} dr\frac{1 - \sqrt{1 - \frac{1}{B^4}(r^4 - r_0^4)}}{\sqrt{r^4 - r_0^4}}\right], \tag{2.24}$$

where $B^2 = \alpha' E/mR$ and $r_1 = B(1 + r_0^4/B^4)^{1/4}$.

For large B ($B/r_0 \gg 1$) the first integral (2.24) becomes

$$\int_{r_0}^{r_1}\frac{dr}{\sqrt{r^4 - r_0^4}} = \frac{1}{4}\pi^{1/2}\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}\frac{1}{r_0} - \frac{1}{B} + O\left(\frac{1}{B^2}\right), \tag{2.25}$$

while for the second one, setting $y = B^{-1}r$, we find

$$\frac{1}{B}\int_{\frac{r_0}{B}}^{1}\frac{dy}{\sqrt{y^4 - B^{-4}r_0^4}}\left(1 - \sqrt{1 - y^4 + r_0^4/B^4}\right), \tag{2.26}$$

which leads to

$$\frac{1}{B}\int_0^1 \frac{dy}{y^2}\left(1-\sqrt{1-y^4}\right) = \frac{1}{B}\left(-1+\frac{(2\pi)^{3/2}}{\Gamma(\frac{1}{4})^2}\right). \tag{2.27}$$

Therefore we find the following quantization condition at large-B limit

$$\left(n+\frac{1}{2}\right)\pi \approx \frac{1}{4}\pi^{1/2}\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}\frac{E}{r_0} - \frac{(2\pi)^{3/2}}{\Gamma(\frac{1}{4})^2}\left(\frac{mR}{\alpha'}\right)^{1/2}\sqrt{E}\,, \tag{2.28}$$

which can be inverted to find the energy as a function of n

$$E \approx 4\,\pi^{1/2}\frac{\Gamma(3/4)}{\Gamma(1/4)} r_0\, n\left[1+8^{3/2}\pi^{3/4}\frac{\Gamma(3/4)^{3/2}}{\Gamma(1/4)^{7/2}}\left(\frac{mRr_0}{\alpha' n}\right)^{1/2}\right]. \tag{2.29}$$

Note that the energy as a function of n has the same form as that in the previous cases for both conformal and finite temperature models, where $\sqrt{M/R}$ plays the role of r_0. Therefore one might conclude that the n dependence of energy has a universal form. Of course, as we will see, it does depend on the dimension of the theory.

3. Circular string in gravity dual of QCD

In the context of AdS/CFT correspondence the gravity dual of the QCD has been proposed in [20] and further studied in [23]. The corresponding gravity solution is obtained from near-extremal D4-brane solution [22] which can be recast to

$$\begin{aligned} ds^2 &= r^{3/2}\left(R^2\,h(r)\,d\phi^2 + d\vec{y}_3^{\,2} - dt^2\right) + \frac{dr^2}{r^{3/2}h(r)} + r^{1/2}d\Omega_4^2\,, \\ h(r) &= 1-\left(\frac{r_0}{r}\right)^3. \end{aligned} \tag{3.1}$$

This background has been used to study several properties of pure four-dimensional Yang-Mills theory including quark-anitquark potential, glueballs masses etc. [23]–[28]. These results are qualitatively in agreement with what is expected from QCD, though it is, by now, known that in the low energy this background gives KK spectrums with the same masses as the glueballs masses and therefore has more states than a pure gauge theory [29]. Nevertheless using this background we would expect to get, at least, some qualitative results by studying semiclassical string in this background. The rotating string in this background has recently been studied in [13]. We shall study a pulsating string in this background which is also wrapped around ϕ direction. This string configuration is given by

$$t=\tau\,, \qquad r=r(\tau)\,, \qquad \phi=m\sigma\,. \tag{3.2}$$

The Nambu-Goto action (2.7) for this solution reads

$$S = \frac{-mR}{\alpha'}\int dt\sqrt{r^3h-\dot{r}^2}\,. \tag{3.3}$$

By making use of a change of variable such that $dr/d\xi = \sqrt{r^3 - r_0^3}$, the hamiltonian can be written as following:

$$H = \sqrt{\Pi^2 + \left(\frac{mR}{\alpha'}\right)^2 \left(\frac{dr}{d\xi}\right)^2}, \tag{3.4}$$

which leads to the following quantization condition in the original r coordinate due to the Bohr-Sommerfeld analysis

$$\left(n + \frac{1}{2}\right)\pi = E\left[\int_{r_0}^{r_1} \frac{dr}{\sqrt{r^3 - r_0^3}} - \int_{r_0}^{r_1} dr \frac{1 - \sqrt{1 - \frac{1}{B^3}(r^3 - r_0^3)}}{\sqrt{r^3 - r_0^3}}\right], \tag{3.5}$$

where $B^{3/2} = \alpha' E/mR$ and $r_1 = B(1 + r_0^3/B^3)^{1/3}$.

One can perform the integrals in (3.5) to find the dependence of energy on n. Doing so we get

$$E \approx 3\sqrt{\pi}\frac{\Gamma(2/3)}{\Gamma(1/6)}\sqrt{r_0 n^2}\left[1 + \frac{1}{\pi^{1/6}}\left(\frac{3\Gamma(2/3)}{\Gamma(1/6)}\right)^{2/3}\left(\frac{mRr_0}{n\alpha'}\right)^{1/3}\right]. \tag{3.6}$$

We note that although the n dependence of the energy is not the same as the previous case, it belongs to another universality class. This can be understood from the fact that the corresponding gravity background (3.1) can be obtained from the AdS_7 black hole compactified on a circle. Therefore we would expect to find a universal form for the energy as a function of n with the same form as that which could have been obtained from circular pulsating membrane in the M-theory on $AdS_7 \times S^4$. To see this, in next section we shall study this membrane configuration in M-theory.

4. Circular membrane in M-theory $AdS_7 \times S^4$ background

In this section we study multiwrapped circular membrane pulsating in the radial direction of AdS_7 in M-theory. Let us start with the gravity solution of $AdS_7 \times S^4$ in the global coordinates

$$\begin{aligned} l_p^{-2} dS^2 &= 4R^2\Big[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho\left(d\psi_1^2 + \cos^2\psi_1 d\psi_2^2 + \sin^2\psi_1 d\Omega_3^2\right) + \\ &\qquad + \frac{1}{4}\left(d\alpha^2 + \cos^2\alpha\, d\theta^2 + \sin^2\alpha\left(d\beta^2 + \cos^2\beta d\gamma^2\right)\right)\Big], \\ d\Omega_3^2 &= d\psi_3^2 + \cos^2\psi_3 d\psi_4^2 + \cos^2\psi_3\cos^2\psi_4 d\psi_5^2\,, \end{aligned} \tag{4.1}$$

where $R^3 = \pi N$. Note that, we are using a unit in which R is dimensionless. The supersymmetric action of the M-theory supermembrane has been studied in [30]. Here we shall only consider the bosonic part of the action which can be written as following:

$$I = -\frac{1}{(2\pi)^2 l_p^3}\int d\xi^3\left(\sqrt{-\det\left(G_{\mu\nu}\partial_a x^\mu \partial_b x^\nu\right)} + \frac{1}{6}\epsilon^{ijk}\partial_i x^\mu \partial_j x^\nu \partial_k x^\lambda C_{\mu\nu\lambda}\right), \tag{4.2}$$

where$(\xi_1, \xi_2, \xi_3) = (\tau, \delta, \sigma)$ are coordinates which parameterize the membrane worldvolume. $x^\mu, \mu = 0, \ldots, 10$ are space-time coordinates and $C_{\mu\nu\lambda}$ is the massless M-theory three-form.

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The rotating membrane solution in this background has been studied in [8, 11]. We now look for a soliton solution representing a multiwrapped membrane pulsating in the radial coordinate which is fixed to a point on S^4. The solution is given by

$$t = \tau\,, \qquad \psi_2 = \sqrt{2}a\delta\,, \qquad \psi_5 = \sqrt{2}m\sigma\,, \qquad \rho = \rho(\tau)\,, \qquad \psi_1 = \frac{\pi}{4}\,, \tag{4.3}$$

all other coordinates are set to zero. For the solution (4.3) the CS part of the membrane action (4.2) is zero and therefore the membrane action (4.2) reads

$$I = -(2R)^3 am \int dt \sinh^2\rho\sqrt{\cosh^2\rho - \dot{\rho}^2}\,. \tag{4.4}$$

Introducing new variable as $\xi = \arcsin(\tanh\rho)$, the action (4.4) gets the following form:

$$I = -(2R)^3 am \int dt \tan^2\xi \sec\xi\sqrt{1-\dot{\xi}^2}\,. \tag{4.5}$$

In these coordinates the hamiltonian is

$$H = \Pi\dot{\xi} - L = \sqrt{\Pi^2 + (2R)^6 a^2 m^2 \tan^4\xi \sec^2\xi}\,, \tag{4.6}$$

which can be treated as a one-dimensional quantum mechanical system such that the potential for H^2 is given by

$$V(\xi) = 2(2R)^6 a^2 m^2 \tan^4\xi \sec^2\xi\,. \tag{4.7}$$

To find the energy levels of the string states one can make use of the Bohr-Sommerfeld analysis. Since ξ has to be positive we can symmetrize the potential and only consider the even states. Thus one gets

$$\left(2n + \frac{1}{2}\right)\pi = \int_{-\xi_0}^{\xi_0} d\xi\sqrt{E^2 - (2R)^6 a^2 m^2 \tan^4\xi \sec^2\xi}\,, \tag{4.8}$$

where $\pm\xi_0$ are the turning points which solve the following equation:

$$E^2 = (2R)^6 a^2 m^2 \tan^4\xi_0 \sec^2\xi_0\,. \tag{4.9}$$

Defining $B = E/(2R)^6 a^2 m^2$ and setting $\tan\xi = B^{1/3}y$, the eq. (4.8) can be recast to

$$\begin{aligned}\left(2n + \frac{1}{2}\right)\pi &= 2(2R)^3 am B^{2/3} \times \\ &\times \left[\int_0^{y_0} \frac{dy}{(B^{-2/3} + y^2)} - \int_0^{y_0} dy \frac{1 - \sqrt{1 - y^4/B^{2/3} - y^6}}{B^{-2/3} + y^2}\right],\end{aligned} \tag{4.10}$$

where y_0 is the turning point in the new coordinates. In the limit of $B \to \infty$ and for large energies, we get

$$E \approx 2n + \frac{1.12}{\pi} 2^{8/3} R (n\sqrt{am})^{2/3}\,, \tag{4.11}$$

which has the same n dependence as that we found in the previous section for the near-extremal D4-brane. This also shows that the n dependence of the energy has a universal form.

Note also that since the gravity solution (4.1) is conjectured to be dual to the $(0,2)$ theory one can use the result (4.11) to study the spectrum of the theory. In fcat, from the $(0,2)$ theory point of view this means that there are operators with following anomalous dimension:

$$\Delta \sim \Delta_0 + R(n\sqrt{am})^{2/3}\,. \tag{4.12}$$

5. Conclusion

In this paper, we have studied new class of semiclassical string solution in the various backgrounds in string theory which are conjectured to be dual to the non-supersymmetric confining gauge theories. This solution represents a multiwrapped circular string pulsating in the radial coordinate of the gravity solution of the near-extremal D3 and D4-branes as well as AdS_5 black hole.

We have evaluated the energy of this string configuration as a function of a large quantum number. We observe that the result is universal, i.e. is the same for both conformal, non conformal and non supersymmetric cases. Of course the result does depend on the dimension of theory. For example for $\mathcal{N}=4$ and its deformations, we find a universal form as following:

$$E \sim n + f(\lambda)\sqrt{mn}\,, \tag{5.1}$$

where λ is 't Hooft effective coupling. We note, however, that their interpretation from different gauge theories point of view might be different. This is because the notion of anomalous dimension is defined only at or near a conformal point. Nevertheless this expression can be thought as the dispersion relation of stationary states in the gauge theory side.

One could also consider those string solutions which pulsate on the internal part of the full string background, e.g. S^5 or S^4. But since the n dependence of energy would have a universal form, we would expect to find the same result as that in the conformal case. For example for deformed AdS_5 background it is given by

$$E \sim 2n + \lambda\frac{m^2}{2n}\,. \tag{5.2}$$

We have also studied a multiwrapped circular membrane pulsating in the radial coordinate of AdS_7 in M-theory. We have then been able to find the energy dependence on a large quantum number n. We note that the n dependence of energy is in the the same universality class as that for the pulsating string in the background of the near-extremal D4-brane. This can be understood from the fact that the near-extremal D4-brane solution can be obtained from AdS_7 black hole compactified on a circle.

On the other hand since the gravity solution (4.1) is dual to the $(0,2)$ theory one can use the result (4.11) to study the spectrum of the theory. In fact, this result is our prediction of existence of an operator in $(0,2)$ theory with anomalous dimension given by (4.12). It would be interesting to find the explicit form of the operator in the $(0,2)$ theory.

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